Exercise 60

Find the absolute maximum and absolute minimum values of f on the given interval.

$$f(x) = xe^{x/2}, \quad [-3, 1]$$

Solution

Take the derivative of the function.

$$\begin{aligned}
f'(x) &= \frac{d}{dx}(xe^{x/2}) \\
&= \left[\frac{d}{dx}(x)\right] e^{x/2} + x\left[\frac{d}{dx}(e^{x/2})\right] \\
&= (1)e^{x/2} + x\left[(e^{x/2}) \cdot \frac{d}{dx}\left(\frac{x}{2}\right)\right] \\
&= e^{x/2} + x\left[(e^{x/2}) \cdot \left(\frac{1}{2}\right)\right] \\
&= \frac{1}{2}(2 + x)e^{x/2}
\end{aligned}$$

Set $f'(x) = 0$ and solve for x.

$$\frac{1}{2}(2 + x)e^{x/2} = 0$$

$$2 + x = 0$$

$$x = -2$$

$x = -2$ is within $[-3, 1]$, so evaluate f here.

$$f(-2) = (-2)e^{(-2)/2} = -\frac{2}{e} \approx -0.735759 \qquad \text{(absolute minimum)}$$

Now evaluate the function at the endpoints of the interval.

$$f(-3) = (-3)e^{(-3)/2} = -\frac{3}{e^{3/2}} \approx -0.66939$$

$$f(1) = (1)e^{1/2} = e^{1/2} \approx 1.64872 \qquad \text{(absolute maximum)}$$

The smallest and largest of these numbers are the absolute minimum and maximum, respectively, over the interval $[-3, 1]$.

The graph of the function below illustrates these results.